The Symmetric Downside-Risk Sharpe Ratio

And the evaluation of great investors and speculators.

William T. Ziemba

The Sharpe ratio is a very useful measure of investment performance. Because it is based on mean-variance theory, and thus is basically valid only for quadratic preferences or normal distributions, skewed investment returns can lead to misleading conclusions. This is especially true for superior investors with many high returns. Superior investors may use capital growth wagering ideas to implement their strategies, which produces higher growth rates but also higher variability of wealth.

My simple modification of the Sharpe ratio to assume that the upside deviation is identical to the downside risk provides a useful modification and gives more realistic results.

Exhibit 1 plots wealth levels using monthly data from December 1985 through March 2000 for the Windsor Fund of George Neff, the Ford Foundation, the Tiger Fund of Julian Robertson, the Quantum Fund of George Soros, and Berkshire Hathaway, the fund run by Warren Buffett, as well as the S&P 500 total return index, U.S. Treasuries, and T-bills. Yearly data are shown in Exhibit 2.

The means, standard deviations, and Sharpe [1966, 1994] ratios of these six funds, based on monthly, quarterly, and yearly net arithmetic and geometric total return data, are shown in Exhibit 3. Shown as well are data on the Harvard endowment (quarterly) plus U.S. Treasuries, T-bills, and U.S. inflation, and number of negative months and quarters.

The first panel in Exhibit 3 shows the data behind

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EXHIBIT 1

Growth of Assets—Monthly Data December 1985–April 2000

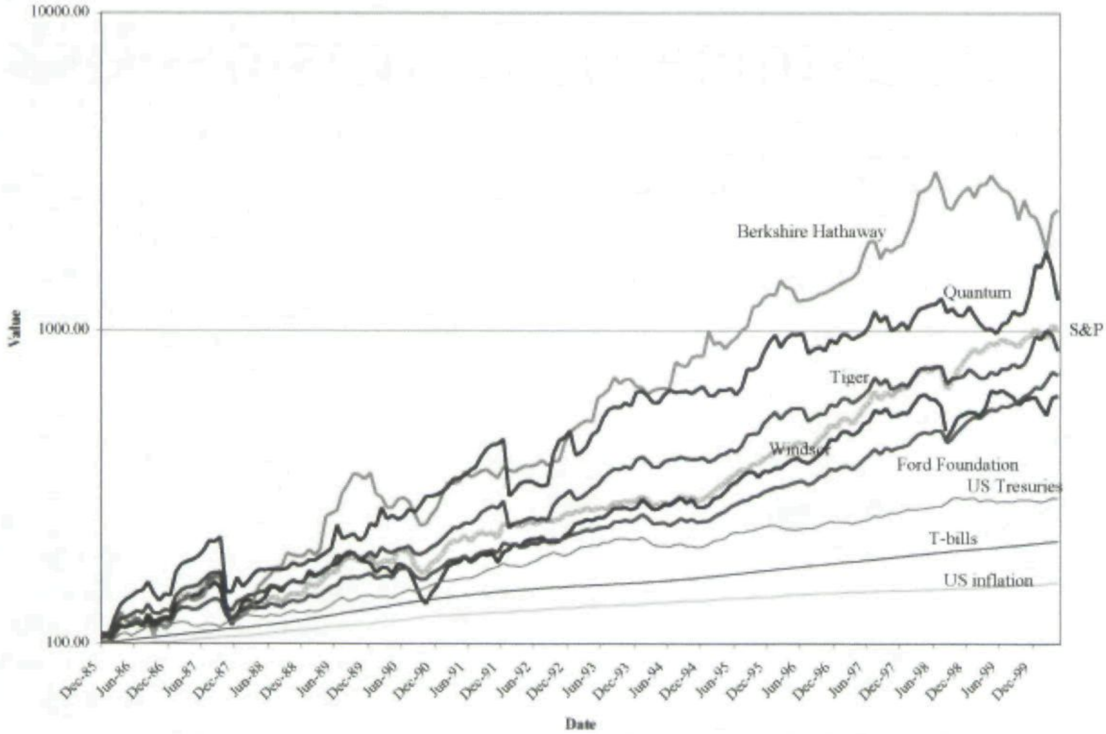


Exhibit 1, which illustrates the high mean returns of Berkshire Hathaway and the Tiger Fund as well as that the Ford Foundation's standard deviation was about a third of Berkshire's. The Ford Foundation actually trailed the S&P 500 mean return. We can see the much lower monthly and quarterly Sharpe ratios compared to the annualized values based on monthly, quarterly, and yearly data.

By the Sharpe ratio, the Harvard endowment and the Ford Foundation had the best performance, followed by the Tiger Fund, the S&P 500 total return index, Berkshire Hathaway, Quantum, Windsor, and U.S. Treasuries. The basic conclusions remain the same according to monthly or quarterly data and arithmetic or geometric means. Because of data smoothing, the Sharpe ratios with yearly data usually exceed those with quarterly data, which in turn exceed the monthly calculations.

The reason for this ranking is that the Ford Foundation and the Harvard endowment, with less growth, also had much less variability. These endowments have different purposes, different investors, different portfolio managers, and different fees, so such differences are not surprising.

Clifford, Kroner, and Siegel [2001] give us similar calculations for a larger group of funds. They also show that over July 1977–March 2000, Berkshire Hathaway's Sharpe ratio was 0.850, Ford's 0.765, and the S&P 500's 0.676. The geometric mean returns were: 32.07% (Buffett), 14.88% (Ford), and 16.71% (S&P 500).

Exhibit 4 shows Buffett's returns on a yearly basis in terms of increase in per share book value of Berkshire Hathaway versus the S&P 500 total return yearly index values for the 40 years 1965–2004. Buffett's geometric and arithmetic means were 22.02% and 22.84%, respectively, versus 10.47% and 11.83% for the S&P 500. This measure does not fully reflect the trading prices of Berkshire Hathaway shares and thus yearly net returns, but it does indicate that Buffett has easily beaten the S&P 500 over these 40 years with a 286,841% increase versus the S&P 500's 5,371% increase.

Typically the Sharpe ratio is computed using arithmetic returns. This is because the basic static theories of portfolio investment management such as mean-variance analysis and the capital asset pricing model are based on arithmetic means. These are static, one-period theories. For asset returns over time, however, the geometric mean is a more accurate measure of average performance because the arithmetic mean is biased upward.

The geometric mean helps mitigate the autocorrelated and time-varying mean and other statistical proper-

EXHIBIT 2
Yearly Return Data (%)

Date	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford	Harvard	S&P 500 Total	US Trea	US T-bills	US Infl
Yearly data, 14 years										
Neg years	2	2	1	0	1	1	1	2	0	0
Dec - 86	20.27	14.17	42.12	26.83	18.09	22.16	18.47	15.14	6.16	1.13
Dec - 87	1.23	4.61	14.13	7.28	5.20	12.46	5.23	2.90	5.47	4.41
Dec - 88	28.69	59.32	10.13	15.76	10.42	12.68	16.81	6.10	6.35	4.42
Dec - 89	15.02	84.57	35.21	24.72	22.15	15.99	31.49	13.29	8.37	4.65
Dec - 90	-15.50	-23.05	23.80	5.57	1.96	-1.01	-3.17	9.73	7.81	6.11
Dec - 91	28.55	35.58	50.58	37.59	22.92	15.73	30.55	15.46	5.60	3.06
Dec - 92	16.50	29.83	6.37	8.42	5.26	4.88	7.67	7.19	3.51	2.90
Dec - 93	19.37	38.94	33.03	24.91	13.07	21.73	9.99	11.24	2.90	2.75
Dec - 94	-0.15	24.96	3.94	1.71	-1.96	3.71	1.31	-5.14	3.90	2.67
Dec - 95	30.15	57.35	38.98	34.34	26.47	24.99	37.43	16.80	5.60	2.54
Dec - 96	26.36	6.23	-1.50	8.03	15.39	26.47	23.07	2.10	5.21	3.32
Dec - 97	21.98	34.90	17.09	18.79	19.11	20.91	33.36	8.38	5.26	1.70
Dec - 98	0.81	52.17	12.46	11.21	21.39	12.14	28.58	10.21	4.86	1.61
Dec - 99	11.57	-19.86	34.68	27.44	27.59	23.78	21.04	-1.77	4.68	2.68

ties of returns that are not independent and identically distributed. If one has returns of +50% and –50% in two periods, for example, the arithmetic mean is zero, which does not reflect the fact that 100 became 150 and then 75. The geometric mean, which is –13.7%, is the correct measure to use.

For investment returns in the 10%-15% range, the arithmetic returns are about 2 percentage points above the geometric returns. But for higher returns, this approximation is not accurate. Hence, I use geometric means as well as more typical arithmetic means in this article.

Lo [2002] points out that we must take care in making Sharpe ratio estimations when the investment returns are not iid, which they are for the investors I discuss here. For dependent but stationary returns Lo derives a correction of the Sharpe ratios that deflates artificially high values to correct values using an estimation of the correlation of serial returns.

Miller and Gehr [1978] and Knight and Satchell [2005] derive exact statistical properties of the Sharpe ratio with normal and lognormal assets, respectively. The Sharpe ratios are almost always lower when geometric means are used rather than arithmetic means; the difference between these two measures is a function of return volatility. The basic conclusions in this research, such as the relative ranking of the various funds, are the same for the arithmetic and geometric means.

Exhibit 5 shows that the Harvard Investment Company, that great school's endowment, had essentially the same wealth record over time as the Ford Foundation. This conclusion is based on quarterly data, which are all I have on Harvard. Harvard beats Ford by the ordinary Sharpe ratio, but Ford is better by the symmetric downside risk measure I develop later.

Before evaluating positive and negative returns performances of these various funds using the Sharpe ratio and my modified version, it is useful to discuss how these funds got their outstanding but sometimes volatile records.

SOME GREAT INVESTORS

Ideally, we would want to penalize only losses such as those shown in Exhibit 6, while rewarding positive returns. The Sharpe ratio penalizes high-return but volatile records.

In the theory of optimal investment over time, it is not quadratic (the utility function behind the Sharpe ratio) but log that yields the most long-term growth. But the elegant results in the Kelly [1956] criterion, as it is known in the gambling literature, and the capital growth theory, as it is known in the investments literature, as proven rigorously by Breiman [1961] and generalized by Algoet and Cover [1988], are long-run asymptotic results (see Hakansson and Ziemba [1995], Ziemba [2003], and MacLean and Ziemba [2005]).

The Arrow-Pratt absolute risk aversion of the log utility criterion:

$$R_A(w) = \frac{-u''(w)}{u'(w)} = 1/w$$

is essentially zero, where u is the utility function of wealth w, and the primes denote differentiation. Hence, in the

EXHIBIT 3

Fund Return Data—December 1985–April 2000

	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford Found	Harvard	S&P500 Total	US Trea	US T-bills	US Infl
Monthly data, 172 months										
Neg months	61	58	53	56	44	na	56	54	0	13
arith mean, mon	1.17	2.15	1.77	2.02	1.19	na	1.45	0.63	0.44	0.26
st dev, mon	4.70	7.66	7.42	6.24	2.68	na	4.41	1.32	0.12	0.21
Sharpe, mon	0.157	0.223	0.180	0.54	0.80	na	0.230	0.14	50.000	- 0.827
arith mean	14.10	25.77	21.25	24.27	14.29	na	17.44	7.57	5.27	3.14
st dev	16.27	26.54	25.70	21.62	9.30	na	15.28	4.58	0.43	0.74
Sharpe, yr	0.543	0.773	0.622	0.879	0.970	na	0.797	0.50	40.000	- 2.865
geomean, mon	1.06	1.87	1.48	1.83	1.16	na	1.35	0.62	0.44	0.26
geo st dev,mon	4.70	7.67	7.42	6.25	2.69	na	4.41	1.32	0.12	0.21
Sharpe, mon	0.133	0.186	0.140	0.222	0.267	na	0.208	0.13	90.000	- 0.828
geo mean, yr	12.76	22.38	17.76	21.92	13.86	na	16.25	7.47	5.27	3.14
geo st dev, yr	16.27	26.56	25.72	21.63	9.30	na	15.28	4.58	0.43	0.74
Sharpe, yr	0.460	0.644	0.486	0.770	0.924	na	0.719	0.48	20.000	- 2.868
Quarterly data, 57 quarters										
Neg quarters	14	15	16	11	11	11	10	15	0	1
mean, qtly	3.55	6.70	5.70	4.35	3.68	3.86	4.48	1.93	1.32	0.79
st dev, qtly	8.01	14.75	12.67	7.70	4.72	4.72	7.52	2.67	0.36	0.49
mean, yr	14.20	26.81	22.79	17.42	14.71	15.44	17.91	7.73	5.29	3.16
st dev, yr	16.03	29.50	25.33	15.40	9.43	9.45	15.05	5.34	0.73	0.97
Sharpe, yr	0.556	0.729	0.691	0.788	0.999	1.074	0.839	0.45	60.000	- 2.188
geomean, qtly	3.23	5.67	4.94	4.07	3.57	3.75	4.20	1.90	1.32	0.79
geo st dev,qtly	8.02	14.79	12.69	7.70	4.72	4.73	7.53	2.67	0.36	0.49
geo mean, yr	12.90	22.67	19.78	16.28	14.29	15.01	16.80	7.59	5.29	3.16
geo st dev, yr	16.04	29.58	25.38	15.41	9.43	9.45	15.06	5.34	0.73	0.97
Sharpe, yr	0.475	0.588	0.571	0.713	0.954	1.029	0.764	0.43	10.000	- 2.190
Yearly Data, 14 years										
Neg years	2	2	1	0	1	1	1	2	0	0
mean,	14.63	28.55	22.93	18.04	14.79	15.47	18.70	7.97	5.40	3.14
st dev,	13.55	30.34	16.17	11.40	9.38	8.52	12.88	6.59	1.50	1.35
Sharpe, yrly	0.681	0.763	1.084	1.109	1.001	1.181	1.033	0.39	00.000	- 1.673
geom mean	13.83	24.99	21.94	17.54	14.43	15.17	18.04	7.78	5.39	3.13
st dev	13.58	30.57	16.20	11.41	9.39	8.53	12.90	6.59	1.50	1.35
Sharpe	0.621	0.641	1.022	1.064	0.962	1.146	0.981	0.36	20.000	- 1.672

EXHIBIT 4

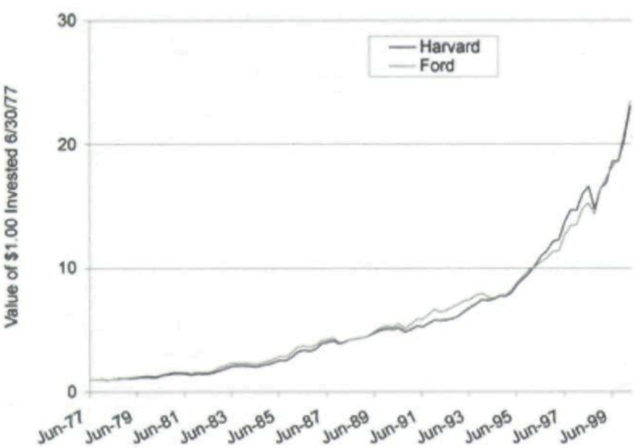
Increase in Per Share Book Value of Berkshire Hathaway versus S&P 500 (dividends included) 1965-2004 (%)

Year	BH	S&P 500	Diff	Year	BH	S&P 500	Diff
1965	23.8	10.0	13.8	1985	48.2	31.6	16.6
1966	20.3	(11.7)	32.0	1986	26.1	18.6	7.5
1967	11.0	30.9	(19.9)	1987	19.5	5.1	14.4
1968	19.0	11.0	8.0	1988	20.1	16.6	3.5
1969	16.2	(8.4)	24.6	1989	44.4	31.7	12.7
1970	12.0	3.9	8.1	1990	7.4	(3.1)	10.5
1971	16.4	14.6	1.8	1991	39.6	30.5	9.1
1972	21.7	18.9	2.8	1992	20.3	7.6	12.7
1973	4.7	(14.8)	19.5	1993	14.3	10.1	4.2
1974	5.5	(26.4)	31.9	1994	13.9	1.3	12.6
1975	21.9	37.2	(15.3)	1995	43.1	37.6	5.5
1976	59.3	23.6	35.7	1996	31.8	23.0	8.8
1977	31.9	(7.4)	39.3	1997	34.1	33.4	.7
1978	24.0	6.4	17.6	1998	48.3	28.6	19.7
1979	35.7	18.2	17.5	1999	.5	21.0	(20.5)
1980	19.3	32.3	(13.0)	2000	6.5	(9.1)	15.6
1981	31.4	(5.0)	36.4	2001	(6.2)	(11.9)	5.7
1982	40.0	21.4	18.6	2002	10.0	(22.1)	32.1
1983	32.3	22.4	9.9	2003	21.0	28.7	(7.7)
1984	13.6	6.1	7.5	2004	10.5	10.9	(0.04)
Overall Gain	286,841	5,371					
Arithmetic Mean	22.84	11.83	11.01				
Geometric Mean	22.02	10.47	10.07				

Sources: Berkshire Hathaway 2004 annual report, Hagstrom [2004].

EXHIBIT 5

Ford Foundation and Harvard Investment Corporation Returns—Quarterly Data June 1977–March 2000



short run, log can be an exceedingly risky utility function with wide swings in wealth values because the optimal bets can be so large.

Long-run exponential growth is equivalent to maximizing the one-period expected log of that period's returns. To illustrate how large Kelly (expected log) bets are, consider the simplest case with Bernoulli trials where you win with probability p and lose with probability $q = 1 - p$.

Log utility is related to negative power utility, namely, for αw^{α} for $\alpha < 0$ since negative power converges to log when $\alpha \to 0$. Kelly [1956] discovered that log utility investors had the best utility function, provided they were very long-run investors. The asymptotic rate of asset growth is

$$G = \lim_{N\to\infty} log\left(\frac{w_N}{w_0}\right)^{\frac{1}{N}}$$

where w_N is period N's wealth and w_0 is initial wealth.

Consider Bernoulli trials that win +1 with probability p and lose −1 with probability $1 - p$. If we win M out of N of these independent trials, the wealth after period N is:

$$w_N = w_0(1 + f)^M(1 - f)^{N-M}$$

where f is the fraction of our wealth bet in each period.

Then:

$$G(f) = \lim_{N\to\infty}\left[\frac{M}{N}log(1 + f) + \frac{N-M}{N}log(1 - f)\right]$$

which by the strong law of large numbers is

$$G(f) = plog(1 + f) + qlog(1 - f) = E(logw)$$

Hence, the criterion of maximizing the long-run exponential rate of asset growth is equivalent to maximizing the one-period expected logarithm of wealth. Hence, to maximize long-run (asymptotic) wealth, maximizing expected log is the way to do it period by period.

The optimal fractional bet, obtained by setting the derivative of $G(f)$ to zero, is $f^* = p - q$, which is simply the investor's edge or expected gain on the bet. If the bets are win $O + 1$ or lose 1—that is, the odds are O to 1 to win—the optimal Kelly bet is $f^* = \frac{p-q}{O}$ or the $\frac{edge}{odds}$. Since edge is a mean concept and odds is a risk concept, you wager more with higher mean and less with higher risk.[1]

In continuous time:

$$f^* = \frac{\mu - r}{\sigma^2} = \frac{edge}{risk(odds)}$$

with optimal growth rate

EXHIBIT 6

Summary—Negative Observations and Arithmetic and Geometric Means

Number of negative	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford Found	Harvard	S&P Total	US Treas
...months out of 172	*61*	58	53	56	44	na	56	54
...quarters out of 57	14	15	*16*	11	10	11	10	15
...years out of 14	*2*	*2*	1	**0**	1	1	1	*2*

$$G^* = \frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 + r = \frac{1}{2}(\text{Sharpe Ratio})^2 + \text{Risk-Free Asset}$$

where μ is the mean portfolio return, r is the risk-free return, and σ^2 is the portfolio return variance. So the ordinary Sharpe ratio determines the optimal growth rate.

Kelly bets can be large. Consider Bernoulli trials where you win 1 or lose 1 with probabilities p and $1 - p$, respectively. Then:

p .5 .51 .6 .8 .9 .99
$1 - p$ .5 .49 .4 .2 .1 .01
f^* 0 .02 .2 .6 .8 .98

So, if the edge is 98%, the optimal bet is 98% of one's fortune. With longer-odds bets, the wagers are lower.

The Kelly bettor is sure to win in the end if the horizon is long enough. Breiman [1960, 1961] was the first to clean up the math in Kelly's [1956] and Latané's [1959] heuristic analyses. He proves that:

$$\lim_{N\to\infty} \frac{w_{KB}(N)}{w_B(N)} \to \infty$$

where $w_{KB}(N)$ and $w_B(N)$ are the wealth levels of the Kelly bettor and another essentially different bettor after N play. That is, the Kelly bettor wins infinitely more than bettor B and moves farther and farther ahead as the long time horizon becomes more distant.[2]

Breiman also shows that the expected time to reach a preassigned goal is asymptotically the shortest with an expected log strategy. Moreover, the ratio of the expected log bettor's fortune to that of any other essentially different investor goes to infinity. The log investor gets all the money in the end if one plays forever.

Hensel and Ziemba [2000] calculate that over 1942 through 1997 a 100% long investor solely in large-cap stocks under Republican administrations and solely in small-cap stocks under Democrats had 24.5 times as much wealth as a 60-40 large-cap/bond investor. That's the idea, more or less.

Keynes and Buffett are essentially Kelly bettors. Kelly bettors will have bumpy investment paths, but most of the time, in the end, accumulate more money than other investors.

Ziemba and Hausch [1986] perform a simulation to show *medium*-run properties of log utility and half-Kelly betting (using the $-w^{-1}$ utility function). Starting with an initial wealth of $1,000 and considering 700 independent wagers with probability of winning 0.19 to 0.57, all with expected values of $1.14 per dollar wagered, they compute the final wealth profiles over 1,000 simulations. These wagers correspond to odds of 1-1, 2-1, 3-1, 4-1, and 5-1.

The results show that the log bettor has more than 100 times initial wealth 16.6% of the time and more than 50 times as much 30.2% of the time. This demonstrates the great power of log betting, as the half-Kelly strategy has very few such high outcomes. Yet this high return comes at a price. Despite making 700 independent bets, each with a 14% success rate, the investor could have lost more than 98% of the initial wealth of $1,000.

Exhibit 7 provides the simulation results. The Ziemba-Hausch simulation used the data in Exhibit 8. The edge over odds gives f^* equal to between 0.14 and 0.028 for the optimal Kelly wagers for 1-1 versus 5-1 odds bets.

The 18 in the first column in Exhibit 7 shows it is possible for a Kelly bettor to make 700 independent wagers, all with a 14% edge, having 19% to 57% chance of winning each wager, and still lose over 98% of one's wealth. Even with half-Kelly, the minimum starting with $1,000 was $145, or a 85.5% loss. This shows the effect of a sequence of very bad scenarios that may be unlikely but are certainly possible.

The last column in Exhibit 7 shows that 16.6% of the time the Kelly bettor increases initial wealth more than 100-fold. The half-Kelly strategy is much safer, as the chance of being ahead after the 700 wagers is 95.4% versus 87.0% for full-Kelly. But the growth rate is much lower, since the 16.6% chance of making 100 times initial wealth is only 0.1% for half-Kelly wagerers. The Kelly bettor accumulates more wealth but with a much riskier time path of wealth accumulation. The Kelly bettor can take a long time to get ahead of another bettor.

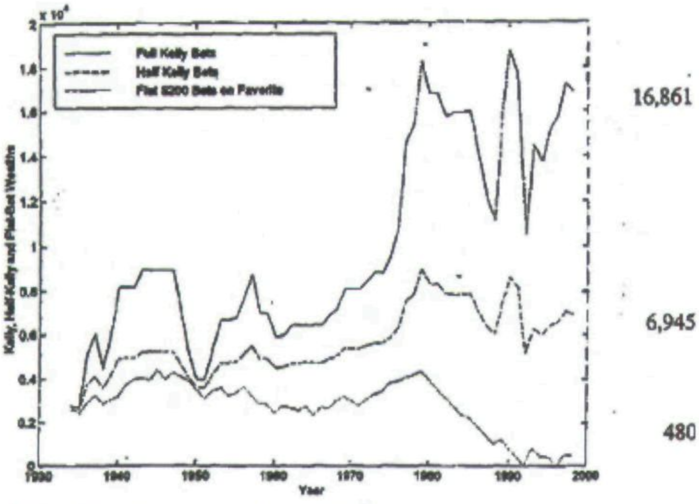
Exhibit 7
Distributions of Final Wealth—Kelly and Half-Kelly Wagers

Final Wealth Strategy	Min	Max	Mean	Median	Number of Times Final Wealth out of 1000 Trials Was >500	>1000	>10,000	>50,000	>100,000
Kelly	18	483,883	48,135	17,269	916	870	598	302	166
Half-Kelly	145	111,770	13,069	8,043	990	954	480	30	1

Exhibit 8
Value of Odds on Wagers

Probability of Winning	Odds	Probability of Being Chosen in the Simulation at Each Decision Point	Optimal Kelly Bets Fraction of Current Wealth
0.57	1-1	0.1	0.14
0.38	2-1	0.3	0.07
0.285	3-1	0.3	0.047
0.228	4-1	0.2	0.035
0.19	5-1	0.1	0.028

Exhibit 9
Place-and-Show Betting on Kentucky Derby 1934-1994



Source: Bain, Hausch, and Ziemba [2005].

What one wants is to have those swings in the right direction. And who better, it seems, at predicting these swings than Warren Buffett. Chopra and Ziemba [1993] show that, in portfolio problems, errors in estimating means, variances, and covariances influence investment performance roughly in the ratio 20:2:1. When risk aversion is lower, as it is with log, the errors are even greater. In this case, the effect on portfolio performance of mean errors can be 100 times the errors in covariances.

Who then would use a log utility function if it is so risky, or use a toned-down log utility function by mixing the log utility investment fraction of one's wealth with cash? These so-called fractional Kelly strategies are actually mathematically equivalent to negative power utility when the assets are lognormally distributed and approximately equivalent otherwise; see MacLean, Ziemba, and Li [2005].

The difference in wealth paths between Kelly and half-Kelly strategies is illustrated in Exhibit 9, which shows the wealth level histories from place-and-show betting on the Kentucky Derby over 1934-1994 following the DrZ system. This system uses a 4.00 dosage index breeding filter rule with full- and half-Kelly wagering and \$200 flat bets on the favorite, with initial wealth of \$2,500 (*dosage* is a measure of a horse's speed over stamina ratio). Starting with \$2,500, full-Kelly yields a final wealth of \$16,861, while half-Kelly, with a much smoother path, has final wealth of \$6,945. These two strategies for the Kentucky Derby are winning ones compared to the losing strategy of betting on the favorite, which turns the \$2,500 into \$480.

Some rather good investors, including four I have worked with or consulted with, have used the Kelly and fractional Kelly approach to turn humble starts into fortunes of hundreds of millions. One was the world's most successful racetrack bettor; see Benter [1994].

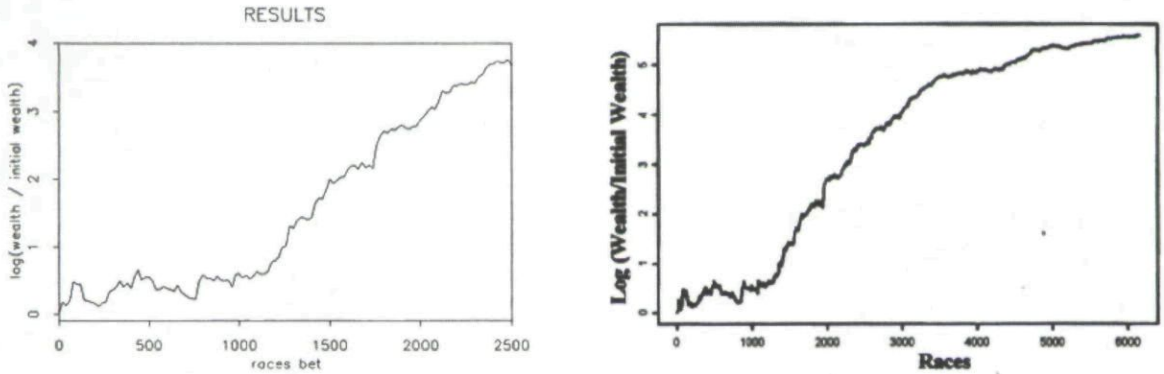
Another was a trend-following futures trader in the Caribbean for whom I designed a Kelly betting system for the 90 liquid futures markets he traded. The Kelly system added \$9 million extra profits per year to his already good betting system based on an ad hoc but sound probability-of-success approach. Another was an options trader eking out nickels and dimes with slightly mispriced options in Chicago.

The fourth was the popularizer of the Kelly approach in sports betting, Edward O. Thorp. Thorp's Princeton-Newport Fund had a net mean return of 15.1%, when the S&P 500's was 10.2% and T-bills returned 8.1%. Interest rates were very high in 1968-1988 while Thorp was running his fund. Thorp had no losing quarters, only three losing months, and a most impressive yearly standard deviation of 4%.

To show the differences between the gamblers and other investors, Exhibit 10 shows Benter's racetrack bet-

EXHIBIT 10

Gamblers Like Smooth Wealth Paths

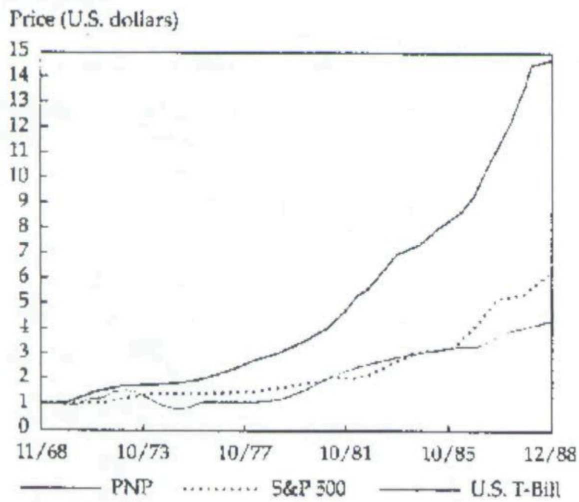


(a) Hong Kong racing syndicate 1989 to 1994. See Benter [1994].

(b) Hong Kong racing syndicate 1989 to 2001 See Benter [2001].

EXHIBIT 11

Princeton-Newport Partners—Cumulative Results November 1968-December 1988



ting record over 1989–1994 and 1989–2001, and Exhibit 11 shows Thorp's record over 1968–1988. Compare these rather smooth graphs with the brilliant but quite volatile record in Exhibit 12 of the eminent economist, John Maynard Keynes, who ran the King's College Chest Fund, the college's endowment, from 1927 until his death in 1945.

Keynes lost more than 50% of his fortune during the difficult years of the depression around the 1929 crash. This bad start was followed by many years of outperformance on relative and absolute terms, so that by 1945 Keynes's geometric mean return was 9.12% versus the U.K. index of -0.89%. Keynes's Sharpe index was 0.385.

The gamblers have several common characteristics:

- They carefully developed anomaly systems with positive means.
- They carefully developed computerized betting systems that automated the betting process.
- They constantly updated their research.
- They were more focused on not losing than on winning in their careful risk control.

Thorp [2006] shows that the Buffett trades are actually very similar to those of a Kelly trader. In Ziemba [2003] I find Keynes is well approximated as a fractional Kelly bettor with 80% Kelly and 20% cash; this is equivalent to the negative power utility function $-w^{-0.25}$. Recall log is when the power coefficient goes to zero, and that log is the most risky utility function that should ever be considered. Positive power utility has less growth and more risk, so is not an acceptable utility function.

SYMMETRIC DOWNSIDE SHARPE RATIO PERFORMANCE MEASURE

Now back to the records of the funds in Exhibit 1. How do we compare these various investors? And can we determine if Warren Buffett really is a better investor than the rather good funds, especially the Ford Foundation and the Harvard endowment?

Exhibit 13 plots the Berkshire Hathaway and Ford Foundation monthly returns as a histogram and shows the losing months and the winning months in a smooth curve. We want to penalize Buffett for losing but not for winning. We define the downside risk as:

EXHIBIT 12
King's College Chest Fund 1927-1945

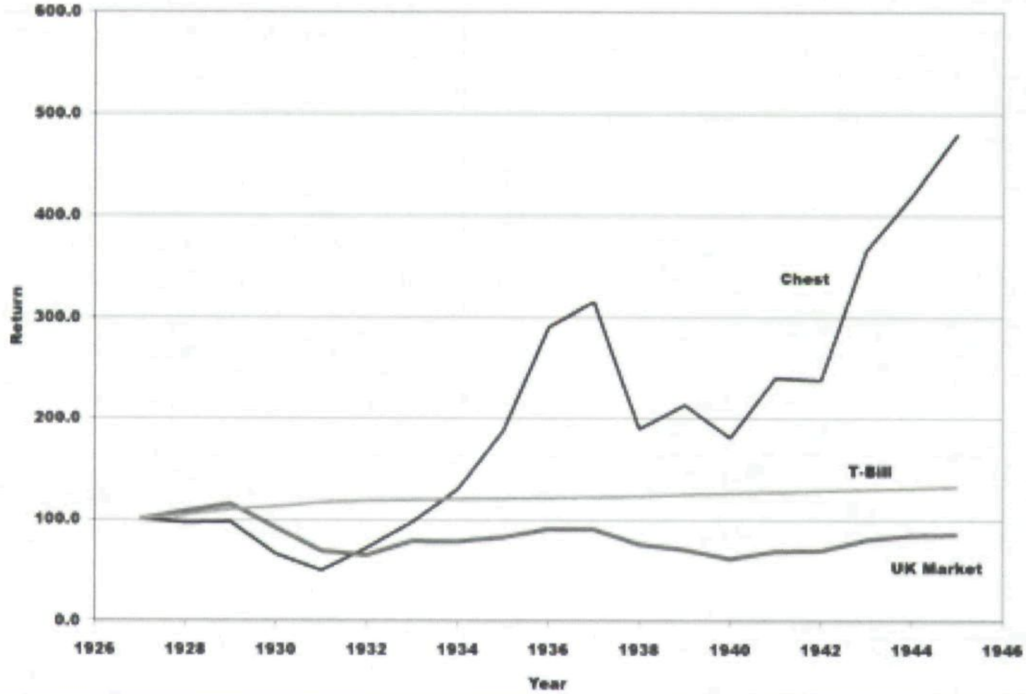


EXHIBIT 13
Berkshire Hathaway versus Ford Foundation—Monthly Returns Distribution January 1977–April 2000

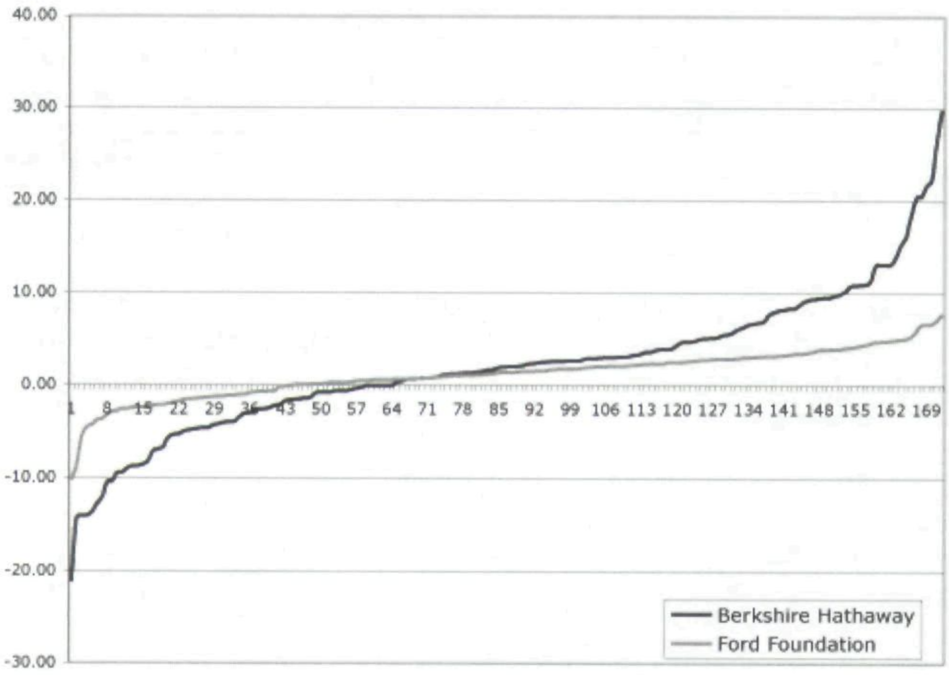


Exhibit 14

Comparison of Ordinary and Symmetric Downside Sharpe Yearly Performance Measures

	Ordinary	Downside
Ford Foundation	0.970	0.920
Tiger Fund	0.879	0.865
S&P500	0.797	0.696
Berkshire Hathaway	0.773	0.917
Quantum	0.622	0.458
Windsor	0.543	0.495

$$\sigma^2_{x_-} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2_-}{n-1}$$

where the benchmark $\bar{x}$ is zero, i is the index on the n months in the sample, and the x_i taken are those below $\bar{x}$, namely, those m of the n months with losses. This is the downside variance measured from zero, not the mean, so it is more precisely the downside risk. To get the total variance, we use twice the downside variance ($2\sigma^2_{x-}$), so that Buffett gets the symmetric gains added, not his actual gains. Using $2\sigma^2_{x_-}$, the usual Sharpe ratio with monthly data and arithmetic returns is:

$$S_- = \frac{\bar{R} - R_F}{\sqrt{2}\sigma_{x-}}$$

Exhibit 14 provides the results for the ordinary and symmetric downside Sharpe ratios using monthly data and arithmetic means.

My measure moves Warren Buffett higher, to 0.917, but not up to the Ford Foundation and not higher because of his high monthly losses. He does gain in the switch from ordinary Sharpe to downside symmetric Sharpe, while all the other funds drop. Ford is 0.920, and Tiger 0.865. The Berkshire Hathaway monthly losses when annualized are over 64% versus under 27% for the Ford Foundation.

Exhibit 14 shows these rather fat tails on the up side and down side of Berkshire Hathaway versus the much less volatile Ford Foundation returns. When Berkshire Hathaway had a losing month, it averaged –5.36% versus +2.15% for all months. Meanwhile, Ford lost 2.44% and won, on average, 1.19%.

Exhibit 15 shows the histogram of quarterly returns for all funds including Harvard (for which monthly data are not available). The plots show that the distributions of all the funds lie between those of Berkshire Hathaway, Harvard, and Ford.

By the quarterly data, the Harvard endowment has a record almost as good as the Ford Foundation's; see Exhibits 16 and 17. Berkshire Hathaway made the most money but also took more risk, and by either the Sharpe or the downside Sharpe measure the Ford Foundation and the Harvard endowment had superior rewards.

I first used this measure in Ziemba and Schwartz [1991] to compare the results of superior investment in Japanese small-cap stocks during the late 1980s. The choice of $\bar{x} = 0$ is convenient and has a good interpretation. But other $\bar{x}$ are possible and might be useful in other applications. This measure is closely related to the Sortino ratio (see Sortino and van der Meer [1991] and Sortino

Exhibit 15

Quarterly Returns Distributions—December 1985–March 2000

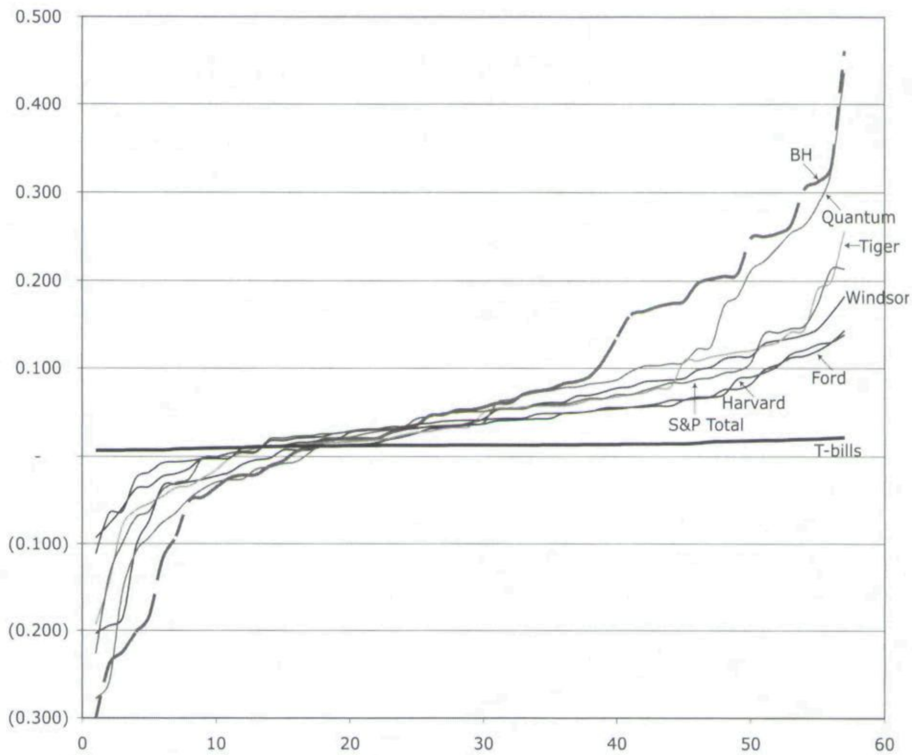


EXHIBIT 16

Yearly Sharpe and Symmetric Downside Sharpe Ratios December 1985–April 2000

	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford Found	Harvard	S&P Total	Trea
qtly data, 57 quarters								
neg qts	14	15	16	11	10	11	10	15
mean, neg	-6.69	-10.50	-7.77	-6.26	-3.59	-2.81	-6.92	-1.35
mean, qtly	3.55	6.70	5.70	4.35	3.68	3.86	4.48	1.93
ds st dev, qtly	10.52	14.00	12.09	8.18	4.44	5.12	9.89	1.35
mean, yr	14.20	26.81	22.79	17.42	14.71	15.44	17.91	7.73
ds st dev, yr	21.04	28.00	24.17	16.35	8.89	10.24	19.78	2.70
ds Sharpe	0.424	0.769	0.724	0.742	1.060	0.991	0.638	0.903
geomean, qtly	3.23	5.67	4.94	4.07	3.57	3.75	4.20	1.90
ds geo st dev,qtly	10.20	13.49	11.80	7.71	3.91	4.99	9.34	1.28
geo mean, yr	12.90	22.67	19.78	16.28	14.29	15.01	16.80	7.59
ds geo st dev, yr	20.41	26.97	23.60	15.43	7.83	9.97	18.68	2.56
ds Sharpe	0.373	0.644	0.614	0.712	1.150	0.975	0.616	0.900

and Price [1994]), which considers downside risk only. That measure does not have the two-sided interpretation of my measure, and the $\sqrt{2}$ does not appear.

The notion of focusing on downside risk is popular these days as it represents real risk better. I started using it in asset-liability models in the 1970s; see Kallberg, White, and Ziemba [1982] and Kusy and Ziemba [1986] for early applications. In those models we measure risk as the downside non-attainment of investment target goals that can be deterministic such as wealth growth over time, or stochastic such as the non-attainment of a portfolio of weighted benchmark returns. See Geyer et al. [2003] for an application of this to the Siemens Austria pension fund.[3]

Calculating the Sharpe ratio and the downside-symmetric Sharpe ratio using quarterly or yearly data does not change the results much, even though it smooths the data because individual monthly losses are combined with gains for lower volatility. The yearly data move us closer to normally distributed returns so the symmetric downside and ordinary Sharpe measures will yield similar rankings.

In Exhibit 2, I showed the yearly returns for the various funds and their Sharpe ratios computed using arithmetic and geometric means and the yearly data. There are insufficient data to compute the downside Sharpe ratios based on yearly data. The Ford Foundation had only one losing year, and that loss was only 1.96%. Berkshire Hathaway had two losing years with losses of 23.1% and 19.9%. The Ford Foundation had a higher Sharpe ratio than Berkshire Hathaway, but that was exceeded by the Tiger and Quantum funds and the S&P 500.

Exhibits 17 and 18 summarize the annualized results using monthly, quarterly, and yearly data. The Ford Foundation had the highest Sharpe ratio, followed by the Harvard endowment, and both of these exceeded Berkshire Hathaway. The Ford Foundation had the highest symmetric-downside Sharpe ratio, followed by Harvard, and both exceeded Berkshire Hathaway and the other funds.

EXHIBIT 17

Summary of Means (%) and Sharpe and Symmetric Downside Sharpe Ratios Annualized

	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford Found	Harvard	S&P Total	Trea
mean (arith, mon)	14.10	25.77	21.25	24.27	14.29	na	17.44	7.57
Sharpe (arith, mon)	0.543	0.773	0.622	0.879	0.970	na	0.797	0.504
ds Sharpe (arith,mon)	0.495	0.917	0.458	0.865	0.920	na	0.696	0.631
mean (geom, mon)	12.76	22.38	17.76	21.92	13.86	na	16.25	7.47
Sharpe (geom, mon)	0.460	0.644	0.486	0.770	0.924	na	0.719	0.482
ds Sharpe (geom, mon)	0.420	0.765	0.358	0.758	0.876	na	0.628	1.053
mean (arith, qtly)	14.20	26.81	22.79	17.42	14.71	15.44	17.91	7.73
Sharpe (arith, qtly)	0.556	0.729	0.691	0.788	0.999	1.074	0.839	0.456
ds Sharpe (arith,qtly)	0.424	0.769	0.724	0.742	1.060	0.991	0.638	0.903
mean (geom, qtly)	12.90	22.67	19.78	16.28	14.29	15.01	16.80	7.59
Sharpe (geom, qlty)	0.475	0.588	0.571	0.713	0.954	1.029	0.764	0.431
ds Sharpe (geom, qtly)	0.373	0.644	0.614	0.712	1.150	0.975	0.616	0.900
mean (arith, yrly)	14.63	28.55	22.93	18.04	14.79	15.47	18.70	7.97
Sharpe (arith, yrly)	0.681	0.763	1.084	1.109	1.001	1.181	1.033	0.390
mean (geom, yrly)	13.83	24.99	21.94	17.54	14.43	15.17	18.04	7.78
Sharpe (geom, yrly)	0.621	0.641	1.022	1.064	0.962	1.146	0.981	0.362

EXHIBIT 18

Summary of Means and Sharpe and Downside Sharpe—Monthly, Quarterly, and Yearly Data Annualized December 1985–March 2000

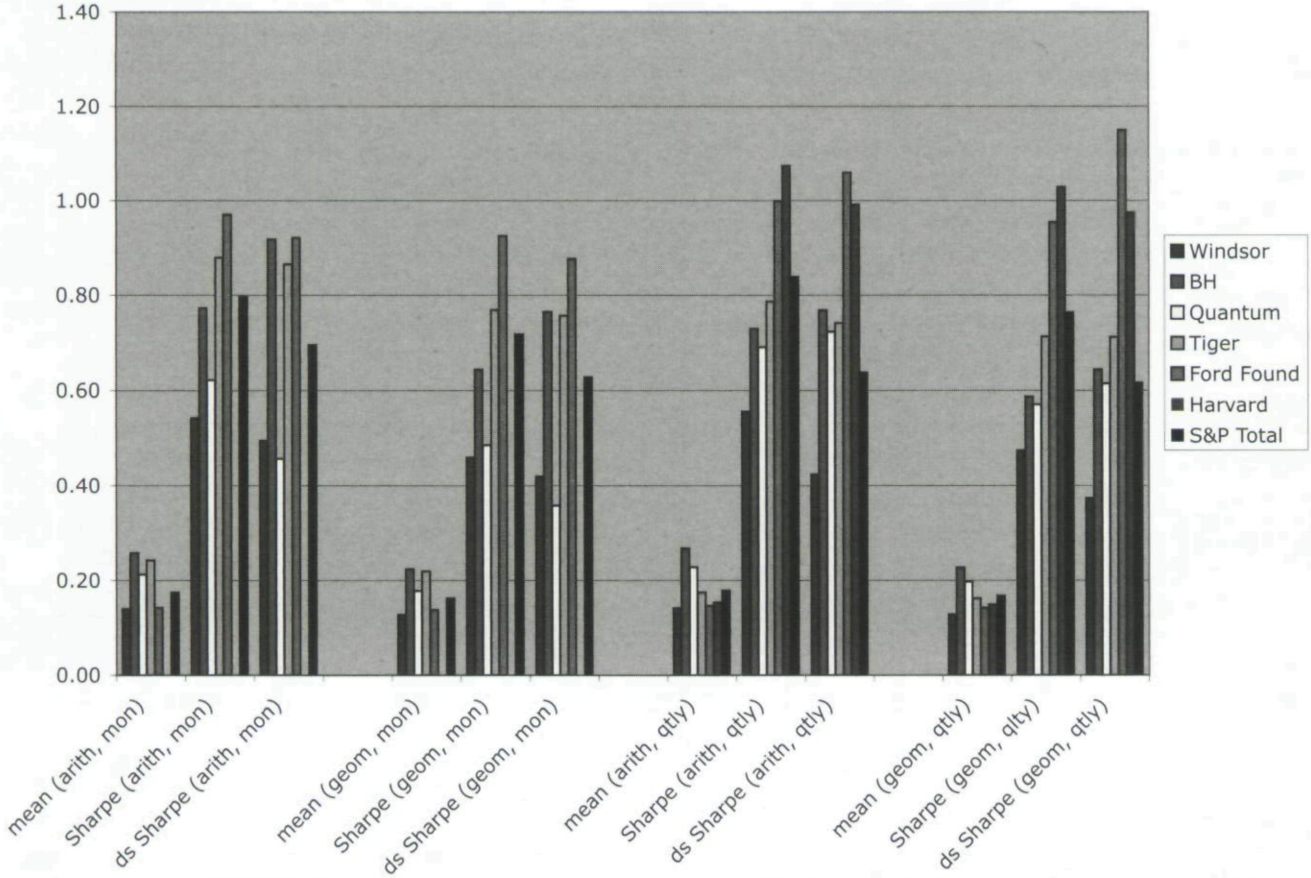


EXHIBIT 19

Asset Allocation of Harvard Endowment—June 2004

Harvard's Holdings By Sector	Annual Returns 1-Year, %	10-Year, %	Weight In Endowment, %
Domestic Equities	22.8	17.8	15
Foreign Equities	36.1	8.5	10
Emerging Markets	6.6	9.7	5
Private Equity	20.8	31.5	13
Hedge Funds	15.7	n.a	12
High Yield	12.4	9.7	5
Commodities	19.7	10.9	13
Real Estate	16.0	15.0	10
Domestic Bonds	9.2	14.9	11
Foreign Bonds	17.4	16.9	5
Inflation-Indexed Bonds	4.2	n.a.	6
Total Endowment	21.1	15.9	105%*

*includes slight leverage

Source: *Barrons*, February 1, 2005.

As Lawrence Siegel of the Ford Foundation privately acknowledges, some of the Ford Foundation's high Sharpe ratio results from dividing by an artificially smoothed standard deviation, due to that institution's high private equity allocation whose market prices do not reflect actual volatility. This is also true of Harvard.

Exhibit 19 shows the Harvard endowment's asset allocation. Its ten-year 15.9% performance on a $22.6 billion portfolio was 3.1 percentage points better than the median of the 25 largest university endowments through June 2004. Harvard has continued its good performance, with extensive use of private equity and other non-traditional investments. Ford's performance in 2001-2002 was poor, and Berkshire Hathaway has doubled in price since the 2000 lows, so the rankings based on more data may well have changed.

Exhibits 6, 16-17, and 20 give a short overview of key data. The numbers in italics indicate the worst outcomes and those in bold the best. Windsor had the most negative months and negative years (tied with Berkshire Hathaway) and the lowest means. Berkshire Hathaway had the highest annual mean returns.

Exhibit 20

Yearly Sharpe and Symmetric Downside Sharpe Ratios December 1985–April 2000 (%)

	Windsor	Berkshire Hathaway	Quantum	Tiger	Ford Found	Harvard	S&P Total	US Trea	T-bills	US Infl
Neg months	61	58	53	56	44	na	56	54	0	13
arith mean, neg	-3.54	-5.36	-5.65	-4.41	-2.24	na	-3.31	-0.87	na	-0.14
arith mean, mon	1.17	2.15	1.77	2.02	1.19	na	1.45	0.63	0.44	0.26
ds st dev, mon	5.15	6.46	10.08	6.34	2.83	na	5.05	1.06	0.00	0.19
arith mean, yr	14.10	25.77	21.25	24.27	14.29	na	17.44	7.57	5.27	3.14
ds st dev, yr	17.85	22.36	34.91	21.97	9.81	na	17.49	3.66	0.00	0.64
ds Sharpe	0.495	0.917	0.458	0.865	0.920	na	0.696	0.631	na	-3.320
geomean, neg	-3.62	-5.48	-5.95	-4.53	-2.26	na	-3.38			
geomean, mon	1.06	1.87	1.48	1.83	1.16	na	1.35	0.62	0.44	0.26
ds geo st dev,mon	5.15	6.46	10.09	6.34	2.83	na	5.05	0.60	0.00	0.00
geo mean, yr	12.76	22.38	17.76	21.92	13.86	na	16.25	7.47	5.27	3.14
ds geo st dev, yr	17.86	22.37	34.94	21.98	9.81	na	17.49	2.09	0.00	0.00
ds Sharpe	0.420	0.765	0.358	0.758	0.876	na	0.628	1.053		

CONCLUSIONS AND CAVEATS

My downside-risk Sharpe ratio measure is ad hoc, as all performance measures are, and adds to but does not close the debate on this subject.

Hodges [1998] proposes a generalized Sharpe measure that eliminates some of the paradoxes the Sharpe measure leads to. It uses a constant, absolute risk-return exponential utility function and general return distributions. When the returns are normally distributed, this is the usual Sharpe ratio, as that portfolio problem is equivalent to a mean-variance model. A better utility function is the constant relative risk aversion negative power.

Leland [1999] shows how to modify βs when there are fat tails into more correct βs in a CAPM framework.

Goetzmann et al. [2002] and Spurgin [2000] show how the Sharpe ratio may be manipulated using option strategies to obtain what looks like a superior record to obtain more funds to manage. Managers sell calls to cut off upside variance, and use the proceeds to buy puts to cut off downside variance, leading to higher Sharpe ratios because of the reduced portfolio variance. These options transactions may actually lead to poorer investment performance in final wealth terms even with their higher Sharpe ratios. Tompkins, Ziemba, and Hodges [2003], for example, show how on average the calls sold and the puts purchased on the S&P 500 both had negative expected values over 1985-2002.

I have not tried to establish when the symmetric downside-risk Sharpe ratio might give misleading results in real investment situations or to establish its mathematical and statistical properties. I note only that it is consistent with an investor whose utility is based on the negative of the disutility of losses. The technique does seem to provide a simple way to avoid penalizing superior performance in order to more fairly evaluate performance.

We will have to find another way to measure and establish the superiority of Warren Buffett. One likely candidate is related to the Kelly approach to evaluate investments, which looks at compounded wealth over a long period of time, which we know at the limit is attained by the log bettor (which Buffet seems to be). After 40 years, most of us believe Buffett is in the skill, not luck, category. After all, \$15 a share in 1965 became \$87,000 in June 2005, but since he is a log bettor only more time will tell.

ENDNOTES

[1]If there are two independent wagers, and the size of the bets does not influence the odds, an analytic expression can be derived; see Thorp [1997, pp. 19-20]. In general, to solve for the optimal wagers when the bets influence the odds, there is dependence. In the case of three or more wagers, one must solve a nonconvex nonlinear program; see Ziemba and Hausch [1984, 1987] for technique. This gives the optimal wager, taking into account the effect of our bets on the odds (prices).

[2]Bettor B must use an essentially different strategy from our Kelly bettor for this to be true. This means that the strategies differ infinitely often. For example, they are the same for the first ten years, and then every second trial is different. This is a technical point to get proofs correct, but nothing much to worry about in practice since non-log strategies will differ infinitely often.

[3]Roy [1952], Markowitz [1959], Mao [1970], Bawa [1975, 1978], Bawa and Lindenberg [1977], Fishburn [1977], Harlow and Rao [1989], and Harlow [1991] have used downside-risk measures in portfolio theories other than those based on mean-variance and related analyses.

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